

Diversity-Multiplexing Tradeoff in MIMO Systems

Dan J. Dechene, *Student Member, IEEE*
 Department of Electrical and Computer Engineering
 The University of Western Ontario
 London, Ontario, N6A 5B9, Canada
 Email: ddechene@uwo.ca

Abstract—It is well known that there exists a tradeoff between the diversity gain and the multiplexing gain achievable by a particular MIMO coding scheme. In this report we first look at the optimal trade-off in the infinite signal-to-noise (SNR) regime of both Rayleigh independent and identically distributed (i.i.d.) channels and a more general MIMO channel model. We then look at the tradeoff of both Space-Time Block Codes (such as Alamouti code) and BLAST Detection schemes. The effect of non-identically distributed and correlated channels is also briefly discussed. Finally, the diversity multiplexing (DIV-MUX) tradeoff is presented under the finite SNR regime.

Index Terms—Diversity, Multiplexing, Space-Time Codes, MIMO, BLAST, Alamouti

I. INTRODUCTION

IN recent years, Multiple-Input, Multiple Output (MIMO) antenna systems have generated a great deal of attention from both the research and industry communities. These systems have been shown [1] that they can improve system performance quite dramatically. To date researchers have utilized MIMO for both diversity approaches to improve error performance [2] or to increase spectral efficiency via spatial multiplexing [3]. In initial research into MIMO systems, these schools of thought were segmented whereas a given scheme would attempt to either improve diversity or increase multiplexing but not both. In more recent works, researchers have focused examining the tradeoffs associated with diversity and multiplexing. The major pioneering work that studied these tradeoffs was by Zheng et. al [4] where the optimal diversity tradeoff in Rayleigh i.i.d. channels was studied.

In this work we look at recent developments in the understanding of the diversity-multiplexing (DIV-MUX) tradeoff with a focus on the initial work by Zheng [4]. The remainder of this project is divided as follows. In the next section we describe a general MIMO system and define diversity gain and multiplexing gain in the context of MIMO systems. In Section III, we examine the optimal trade-off in both Rayleigh and more general

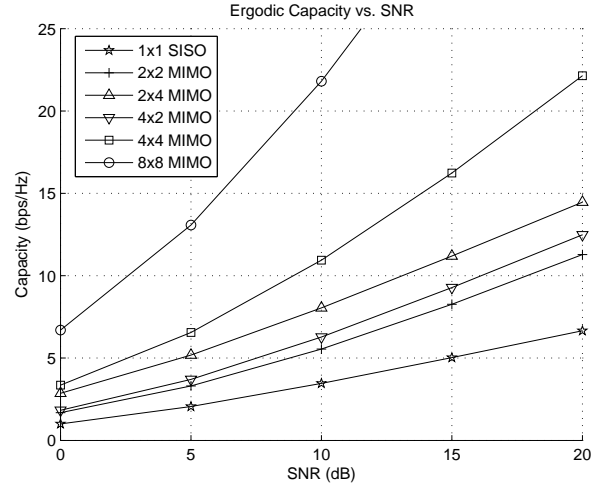


Fig. 1: Ergodic Capacity of MIMO Channel without CSI, Rayleigh i.i.d.

MIMO channels. In Section IV explores the DIV-MUX tradeoff achieved in real schemes as well as relaxing the high SNR assumption in Section V. Finally we draw some conclusions on the DIV-MUX tradeoffs and propose some future directions for this work in Section VI.

II. MIMO

In general, we can represent a flat-fading multiple-input, multiple output (MIMO) antenna system with M_T transmitter antennas and M_R receiver antennas can be represented as [1]

$$\mathbf{r}(t) = \mathbf{H}(t)\mathbf{s}(t) + \mathbf{n}(t) \quad (1)$$

where $\mathbf{r}(t)$ is the $M_R \times 1$ receive vector, $\mathbf{s}(t)$ is the $M_T \times 1$ transmitter vector, $\mathbf{H}(t)$ is the $M_R \times M_T$ channel transfer matrix with entries $h_{i,j}(t)$ representing the transfer function from the j^{th} transmitter antenna to the i^{th} receiver antenna and $\mathbf{n}(t)$ as the $M_R \times 1$ additive noise vector with i.i.d. entries such that $\mathbf{n}(t) \sim \mathcal{CN}(0, N_0 \mathbf{I}_{M_R})$.

The strength of MIMO lies in the methods of pre and post-processing the transmitted and received vectors

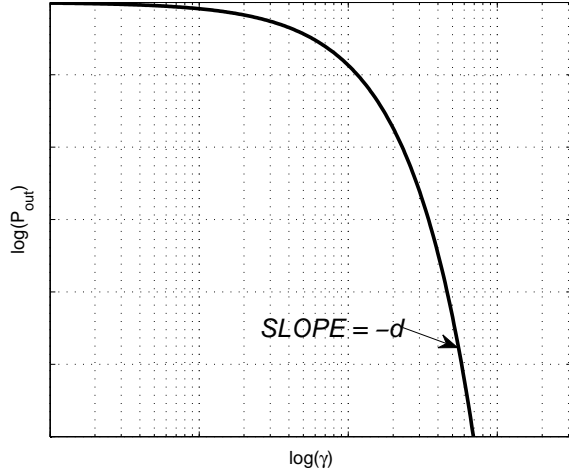


Fig. 2: Diversity Gain Measurement

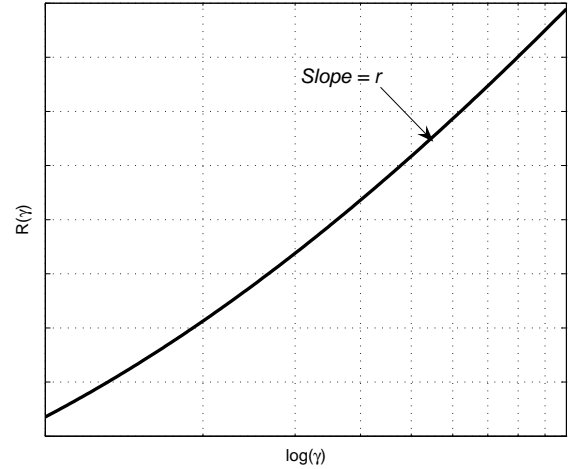


Fig. 3: Multiplexing Gain Measurement

($\mathbf{s}(t)$ and $\mathbf{r}(t)$ respectively) to exploit the fact that the entries of $\mathbf{H}(t)$ are not identical meaning that the transfer function between individual antenna pairs may differ. Codes used for pre-coding determine the ability of a system to achieve either diversity or multiplexing gain, or a tradeoff of both; it is the tradeoffs of these that are discussed in this report.

A. Channel Capacity

The channel capacity of the MIMO channel above is time varying. It is well known [1] that the MIMO channel capacity for an open loop MIMO system (one without channel feedback) is given as

$$C(t) = \log_2 \det \left(I_{M_R} + \frac{\gamma}{M_t} \mathbf{H}(t) \mathbf{H}^\dagger(t) \right) \text{ bps/Hz} \quad (2)$$

The time varying capacity quantity is often of very little use in practice. Therefore there have been two alternative capacity definitions defined: Ergodic and Outage capacities. The ergodic capacity of a MIMO is the expectation over all realizations of the channel such as

$$\bar{C} = \mathbb{E}_{\mathbf{H}} \left[\log_2 \det \left(I_{M_R} + \frac{\gamma}{M_t} \mathbf{H} \mathbf{H}^\dagger \right) \right] \text{ bps/Hz} \quad (3)$$

where the expectation is over the distribution of \mathbf{H} . For various channels (such as Rayleigh i.i.d. [5]) this is well known and is given in Figure 1 for various MIMO configurations.

The outage capacity describes the rate at which reliable transmission can be guaranteed with a certain probability. Denoting ρ is the probability of an outage event, the outage capacity is given as the rate R that

satisfies the below equation.

$$\rho = Pr[C(t) < R] \quad (4)$$

$$= Pr \left[\log_2 \det \left(I_{M_R} + \frac{\gamma}{M_t} \mathbf{H}(t) \mathbf{H}^\dagger(t) \right) < R \right] \quad (5)$$

which also depends on the distribution of the MIMO channel matrix. The above rate R is the $\rho\%$ outage capacity.

B. What is Diversity

Diversity in the context of MIMO systems relates to the ability of a system to improve performance via the differing transfer function between antenna pairs. The concept of diversity gain is a method used to evaluate the diversity of a given scheme. The diversity gain of a system measures the decay of the probability of error with respect to the signal-to-noise ratio (SNR). It is common [4] to measure this quantity as

$$\lim_{\gamma \rightarrow \infty} \frac{\log P_e(\gamma)}{\log \gamma} = -d \quad (6)$$

where $P_e(\gamma)$ is the average probability of error at SNR= γ and d is the diversity gain. Note that this definition is asymptotic and in the high SNR region (where the probability of error decays in a linear fashion). Figure 2 shows how diversity gain is measured. A higher diversity gain ensures a lower probability of error for a given SNR. The result being that more reliable communication can be achieved with a given transmission power, or alternatively transmission power can be reduced to achieve a certain target performance. We have previously presented the definition of the outage capacity. The outage capacity formulation is used by Zheng to derive [4] the

optimal DIV-MUX tradeoff by way of placing bounds on the upper and lower probability of outage versus the probability of error.

C. What is Multiplexing

Multiplexing is used by MIMO systems to increase the system throughput. This is accomplished by transmitting multiple streams of data through the multiple antennas. It is widely discussed that a full rank, richly scattered MIMO channel's capacity approximately scales with the number of antennas. This potential scaling factor describes the gain achievable using a MIMO channel over a SISO channel without increasing channel bandwidth or sacrificing performance and is shown in Figure 3. The spatial multiplexing gain is commonly computed as [4]:

$$\lim_{\gamma \rightarrow \infty} \frac{R(\gamma)}{\log \gamma} = r \quad (7)$$

where $R(\gamma)$ is the data rate at an SNR of γ and r is the spatial multiplexing gain.

III. OPTIMAL DIVERSITY-MULTIPLEXING TRADEOFF

To describe the optimal DIV-MUX tradeoff, consider the MIMO channel discussed in Section II. We observe that both the outage and ergodic capacities depend on the distribution of the channel entries $h_{i,j}$. As such the remainder of this this Section will be divided into two parts. In the first part, we discuss the optimal trade-off in Rayleigh i.i.d. channel as in [4] and the dependance on block length. We will further extend this to general channel distributions in the second part of this Section.

A. Rayleigh Fading MIMO Channel

The optimal DIV-MUX tradeoff was originally described by Zheng in [4]. For simplicity we will not reproduce the entire derivation here as it is quite length, but for brevity we simply state the result and refer the interested reader to Zheng's work in [4] for the entire derivation.

1) *Optimal Tradeoff*: The derivation of the optimal tradeoff is based on the slope of the outage probability as SNR goes to infinity. Alternatively, this occurs when the smallest eigenvalue of the channel matrix \mathbf{H} approaches 0 and therefore is based on the distribution of the eigenvalues of the channel (and therefore resulting channel structure). For brevity, this optimal tradeoff $d^*(r)$ is given as the piecewise function connecting the points $(r, d(r))$ where

$$d(r) = (M_R - r)(M_T - r), \quad r = 0, \dots, M \quad (8)$$

where $M = \min\{M_T, M_R\}$, and the coding blocklength is greater than $M_R + M_T - 1$.

2) *Block Length Dependence*: The capacity expressions in general (such as the one in (2)) assume that infinite code word lengths are used to encode data over the channel. However Zheng has shown [4] that the above optimal tradeoff result is valid for block lengths greater than $M_T + M_R - 1$ for Rayleigh i.i.d. channels. From a trade-off stand point, the performance of infinite codes lengths is achieved with practical block lengths.

B. General Channels

The work by Zheng provided insightful results into the optimal trade-off between diversity and multiplexing in MIMO channels. However this result was limited to the Rayleigh i.i.d. channels. The work by Zhao [6] extends this by applying results from Zheng and expanding this to derive the optimal trade-off in channels with general channel distributions.

1) *Extension*: To describe the extensions by Zhao, we denote the following probability distribution function (pdf)

$$p_{\mathbf{h}}(h) = a|h|^t e^{-b|h-c|^\beta} \quad (9)$$

where $a > 0$, $b > 0$, $\beta > 0$, $t \in \mathcal{R}$, and $c \in \mathcal{C}$ where \mathcal{R} and \mathcal{C} denote the set of all real and complex number respectively. The above equation can be used to describe numerous channel distributions. Common channel model parameters are shown in Table I with additional parameters explained in [6]

TABLE I: Parameters of Various Fading Channels

Channel	PDF	a	b	c	t	β
Rayleigh	$\frac{1}{\pi\Omega} e^{-\frac{ h ^2}{\Omega}}$	$\frac{1}{\pi\Omega}$	$\frac{1}{\Omega}$	0	0	2
Rician	$\frac{1}{\pi\Omega} e^{-\frac{ h-\mu ^2}{\Omega}}$	$\frac{1}{\pi\Omega}$	$\frac{1}{\Omega}$	μ	0	2
Nakagami- m	$\frac{m^m h ^{2m-2}}{\pi\Omega^m \Gamma(m)} e^{-\frac{m h ^2}{\Omega}}$	$\frac{m^m}{\pi\Omega^m \Gamma(m)}$	$\frac{m}{\Omega}$	0	$2m-2$	2
Weibull	$\frac{\alpha\Omega^{-\alpha}}{2\pi} h ^{\alpha-2} e^{-(\frac{ h }{\Omega})^\alpha}$	$\frac{\alpha\Omega^{-\alpha}}{2\pi}$	$\Omega^{-\alpha}$	0	$\alpha-2$	α

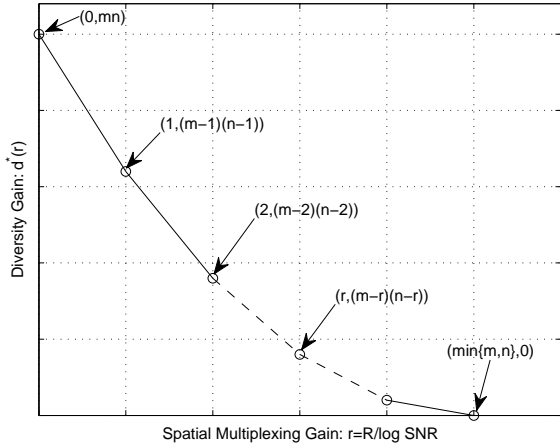


Fig. 4: Optimal Diversity-Multiplexing Tradeoff for Rayleigh i.i.d. Channels

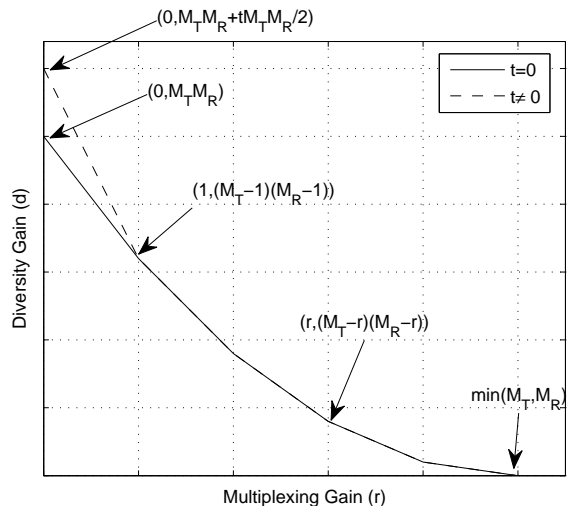


Fig. 5: Optimal Diversity-Multiplexing Tradeoff in General i.i.d. Channels

As in [4], the derivation of the optimal tradeoff is non-trivial and for the sake of brevity we simply present its result. Assuming a large enough block length of l where $l \geq M_T + M_R - 1 + M_T M_R t / 2$, the optimal tradeoff $d^*(r)$ is given as the piecewise function connecting the points below as $(r, d(r))$ where

$$d(r) = \begin{cases} (M_R - r)(M_T - r) & \text{if } r = 1, \dots, M \\ M_R M_T + \frac{t}{2} M_R M_T & \text{if } r = 0 \end{cases} \quad (10)$$

C. Non-Idealities - Correlation and Non-Identical Distributions

We have previously discussed the optimal tradeoff in the case of spatially white, Rayleigh i.i.d. channels. However in practice, there may exist spatial correlation in the MIMO channel matrix or entries may not be identically distributed.

1) *Non-Identically Distributed Channels*: In general, it is possible the individual channel entries $h_{i,j}$ are not identically distributed, but each entry can be generalized by the above model. As previously realized, only the tradeoff region where $r < 1$ differs from that Rayleigh i.i.d. In this case, we can generalize $d(0)$ such that

$$d(0) = M_R M_T + \sum_{i=1}^{M_R} \sum_{j=1}^{M_T} t_{i,j} / 2 \quad (11)$$

2) *Kronecker Correlated Channels*: A common model to incorporate the effect of channel correlation is the Kronecker correlation model. This Kronecker MIMO

channel model is given as [7]

$$\mathbf{H}_{corr} = \frac{1}{\sqrt{\text{tr}\{\mathbf{R}_{R_X}\}}} \mathbf{R}_{R_X}^{1/2} \mathbf{H} (\mathbf{R}_{T_X}^{1/2})^T \quad (12)$$

where \mathbf{R}_{R_X} and \mathbf{R}_{T_X} are the receiver and transmitter spatial correlation matrices and \mathbf{H} is the uncorrelated in space MIMO channel matrix as before.

Under the above correlation model, the optimal diversity multiplexing tradeoff has been shown [6] not to be affected by spatial correlation if the following conditions are met:

- 1) $\text{rank}\{\mathbf{R}_{R_X}\} = M_R$, and
- 2) $\text{rank}\{\mathbf{R}_{T_X}\} = M_T$

i.e., the receiver and transmitter correlation matrices are full rank. This results implies sufficient conditions on the channel correlation structure to maintain the optimal DIV-MUX tradeoff.

D. Observations

In the above subsections we have examined the optimal DIV-MUX tradeoff in both Rayleigh and more general channels. The optimal tradeoff for a Rayleigh i.i.d. MIMO channel is shown in Figure 4 while we contrast that to the tradeoff of a more general i.i.d. MIMO channel model in Figure 5. We make some interesting observations on the optimal tradeoff. Firstly, we observe that the general model yields identical tradeoff to that of the Rayleigh model for a multiplexing gain of 1. From a design standpoint, this means that a broad class of channels yield identical optimal tradeoff in the region of $r \geq 1$. Secondly, the optimal trade-off can be achieved with relatively low blocklengths ($> M_T + M_R - 1 +$

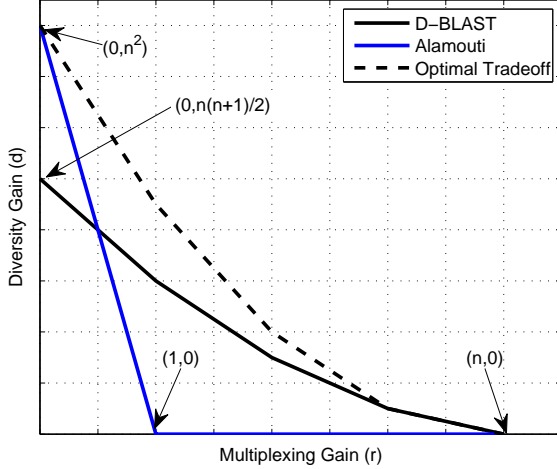


Fig. 6: Optimal Tradeoff of Alamouti and D-BLAST

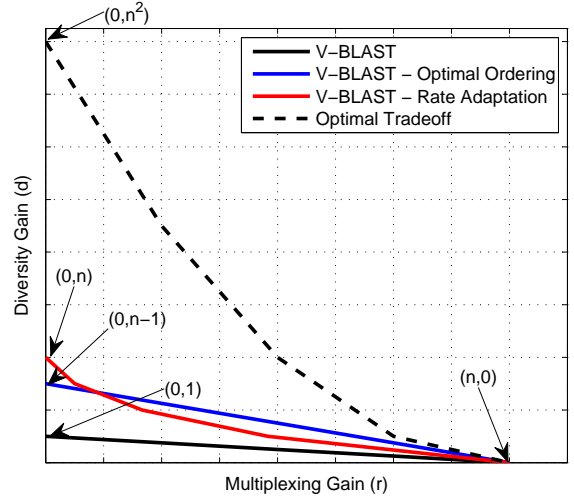


Fig. 7: Optimal Tradeoff of Various V-BLAST Strategies

$M_R M_T t/2$). This is an important characteristic, as this means optimal codes can be designed for delay sensitive applications. Finally we have examined the restrictions on the correlation structure of the channel in maintaining the overall DIV-MUX tradeoff.

IV. DIVERSITY-MULTIPLEXING TRADEOFF OF REAL SCHEMES

In the last section we examined the optimal diversity-multiplexing tradeoff achievable by an optimal coding scheme. In this section we examine the tradeoff achievable by the Alamouti and BLAST detection schemes.

A. Alamouti

Alamouti coding is a block coding scheme where data is coded across multiple antennas to increase the diversity gain. The Alamouti scheme for a 2×2 MIMO antenna system is well known as [2]

$$\mathbf{C} = \begin{bmatrix} s_1 & s_2 \\ -s_2^\dagger & s_1^\dagger \end{bmatrix} \quad (13)$$

where s_1 and s_2 are data symbols. Further, it is well known that these schemes can achieve the full diversity gain. For Rayleigh i.i.d. channels, Zheng showed [4]

$$d_A(r) = M_R M_T (1 - r)^+ \quad (14)$$

where $(x)^+ = \min(0, x)$.

B. Larger Space-Time Block Codes (STBCs)

Although STBCs for larger MIMO systems exists, we note however that Tarokh in [8] proved that the full rate

Alamouti code discussed above is the only code that is full rate, and only Quasi-Orthogonal codes for systems above 2×2 are full rate but sacrifice the maximum diversity tradeoff achievable with the Alamouti scheme.

C. D-BLAST

Diagonal Bell Laboratories Layered Space-Time (D-BLAST) coding designed by Foschini [9] describes a method of being able to achieve both diversity and multiplexing gain. This is accomplished by transmitting data over multiple streams in a diagonal fashion which allows symbol detection and cancellation. A simple transmit configuration for 4×4 D-BLAST is given as

$$\mathbf{C} = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 & \cdots \\ s_0 & s_1 & s_2 & s_3 & \cdots \\ s_{-1} & s_0 & s_1 & s_2 & \cdots \\ s_{-2} & s_{-1} & s_0 & s_1 & \cdots \end{bmatrix} \quad (15)$$

where we can clearly observe the matching symbols on the time diagonal. Assuming a square MIMO channel ($n = M_R = M_T$), the DIV-MUX tradeoff achievable by D-BLAST then is given by [4] the curve connecting $(r, d_D(r))$ where

$$d_D(r) = \frac{(n-r)(n-r+1)}{2} \quad (16)$$

D. V-BLAST

Vertical BLAST (or V-BLAST) [3] is extension of D-BLAST designed by Wolniansky et. al. V-BLAST employs a simpler detection scheme relative to D-BLAST by not employing inter-stream coding. A simple V-BLAST scheme employed in a 4×4 MIMO system can

be given as

$$\mathbf{C} = \begin{bmatrix} s_1 & s_5 & s_9 & s_{13} & \cdots \\ s_2 & s_6 & s_{10} & s_{14} & \cdots \\ s_3 & s_7 & s_{11} & s_{15} & \cdots \\ s_4 & s_8 & s_{12} & s_{16} & \cdots \end{bmatrix} \quad (17)$$

The design of V-BLAST results in a much more simplified detection scheme than D-BLAST which is why it has attracted significant attention in recent years. V-BLAST detection is an iterative process wherein symbols from each transmitter antenna are detected and subsequently canceled during each symbol interval.

Without any consideration for ordering of detected symbols, the DIV-MUX tradeoff achievable by V-BLAST is given as [4]

$$d_V(r) = \left(1 - \frac{r}{n}\right)^+ \quad (18)$$

1) *V-BLAST with Optimal Ordering*: In V-BLAST, symbols are sequentially detected and canceled to decode an entire set of symbols. It was first noted in [3] that by dynamically choosing the detection order such that at each stage of detection, the post-processing SNR is maximized, V-BLAST performance can be improved. Pseudocode for optimal detection ordering for V-BLAST can be found in Algorithm 1 where $*$ denotes the pseudoinverse, $\mathcal{Q}(\cdot)$ denotes quantized symbol selection and $\mathbf{H}_{\overline{k_i}}$ denotes the matrix where the columns k_1, \dots, k_i have been zeroed out. Optimal ordering has been shown [4] to provide an upper bound on the DIV-MUX tradeoff such that

$$d_{V,ordered}(r) \leq (n-1) \left(1 - \frac{r}{n}\right) \quad (19)$$

2) *V-BLAST with Per-Stream Rate Adaptation*: In lieu of optimally ordering the independent streams, a designer may vary the rate allocated to each substream to achieve and overall multiplexing gain. The DIV-MUX tradeoff in this case can be given as the piecewise function [4] connecting $(r_k, n-k)$ where

$$r_k = \sum_{i=0}^{k-1} \frac{k-i}{n-i} \quad (20)$$

E. Observations

The performance of both the Alamouti scheme and the D-BLAST scheme is shown in Figure 6 along with the optimal DIV-MUX tradeoff. We observe that the Alamouti scheme can achieve the full diversity gain offered by the channel, however can only achieve a maximal diversity gain of 1. D-BLAST on the other hand provides the ability to provide the full multiplexing

Algorithm 1 Zero-Forcing V-BLAST Detection with Ordering

Initialization:

- 1: $i \leftarrow 1$
- 2: $\mathbf{r}_1 = \mathbf{r}$
- 3: $\mathbf{G}_1 = \mathbf{H}^*$
- 4: $k_1 = \arg \min_j \|(\mathbf{G}_1)_{j'}\|^2$

Recursion while $i \leq N_T$:

- 5: $\mathbf{w}_{k_i} = (\mathbf{G}_i)_{k_i}$
 - 6: $y_{k_i} = \mathbf{w}_{k_i}^T \mathbf{r}_i$
 - 7: $\widehat{a}_{k_i} = \mathcal{Q}(y_{k_i})$
 - 8: $\mathbf{r}_{i+1} = \mathbf{r}_i - \widehat{a}_{k_i} (\mathbf{H})_{k_i}$
 - 9: $\mathbf{G}_{i+1} = \mathbf{H}_{\overline{k_i}}^*$
 - 10: $k_{i+1} = \arg \min_{j' \notin \{k_1, \dots, k_i\}} \|(\mathbf{G}_{i+1})_{j'}\|^2$
 - 11: $i \leftarrow i + 1$
-

gain while it can only provide a fraction of the overall maximum diversity gain.

In Figure 7 we demonstrate the tradeoff of V-BLAST without ordering, with optimal ordering and with rate adaptation respectively. All three schemes can achieve the full multiplexing gain, however only the rate adaptation scheme can provide upwards of a diversity gain equal to the number of antennas n . We note that all 3 V-BLAST detection approaches fall short of the performance of D-BLAST shown in Figure 6. This tradeoff is a result of the lack of inter-stream encoding in V-BLAST and is afforded in practice due to the decreased implementation complexity in V-BLAST detection compared to D-BLAST.

V. DIVERSITY-MULTIPLEXING TRADEOFF IN FINITE SNR

The optimal DIV-MUX tradeoff characterized by Zheng and further by Zhao provided insight into the tradeoff in the infinite SNR regime. In practice however, wireless communication systems do not operate in this region. The work by Narasimhan et. al. [10] looks at the finite SNR tradeoff for both orthogonal space-time block codes (OSTBCs) and with horizontally encoded spatial multiplexing (SM-HE) codes under Rayleigh i.i.d. conditions. OSTBCs are a set of codes that include the Alamouti scheme while SM-HEs encompass the V-BLAST detection scheme.

To describe the performance we define a new definition of both diversity and multiplexing gain as

$$r(\rho) = \frac{R}{\log_2(1 + \rho)} \quad (21)$$

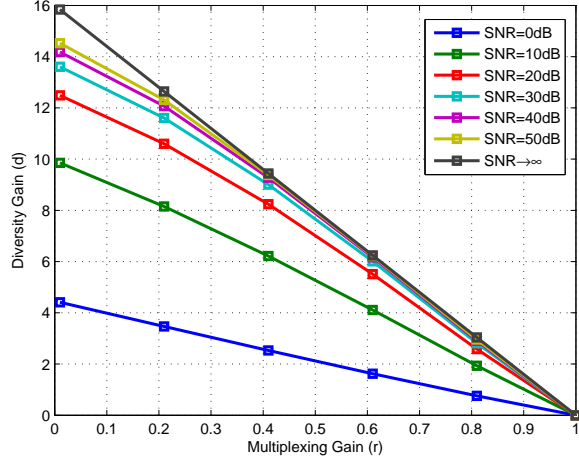


Fig. 8: Finite SNR Tradeoff of Alamouti Code

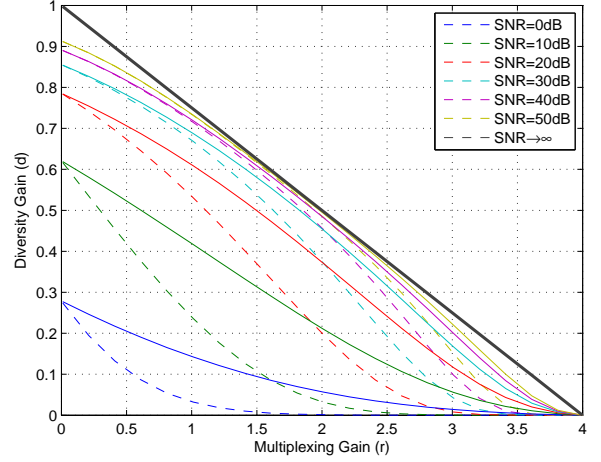


Fig. 9: Finite SNR Tradeoff of V-BLAST

and

$$d(r, \rho) = -\frac{\rho}{P_{out}(r, \rho)} \cdot \frac{\partial P_{out}(r, \rho)}{\partial \rho} \quad (22)$$

where the diversity and multiplexing gain is a function of SNR ρ .

A. Alamouti

Based on the above definition, it is sufficient to know the outage probability for a given SNR and multiplexing gain r . Firstly, we note that the instantaneous capacity of a MIMO channel with rate 1 Alamouti code is

$$C(t) = \log_2(1 + \frac{\rho}{M_T} \|\mathbf{H}(t)\|_F^2) \quad (23)$$

where $\|\cdot\|_F^2$ is the Frobenius norm operation. The outage probability can then be described as [1]

$$P_{out}(r, \rho) = \Pr[C(t) < r \log_2(1 + \rho)] \quad (24)$$

$$= \frac{\Gamma(\frac{M_T}{\rho} [(1 + \rho)^r - 1], M_T M_R)}{(M_T M_R - 1)!} \quad (25)$$

where $\Gamma(x, a)$ is the incomplete gamma function such that

$$\Gamma(x, a) = \int_0^x t^{a-1} e^{-t} dt \quad (26)$$

Finally by using (22) and (25) one can obtain the diversity tradeoff for Alamouti code as given in (27).

$$d_A(r, \rho) = \frac{\frac{M_T}{\rho} (\frac{M_T}{\rho} [(1 + \rho)^r - 1])^{M_T M_R - 1}}{\Gamma(\frac{M_T}{\rho} [(1 + \rho)^r - 1], M_T M_R)} \exp\left(-\frac{M_T}{\rho} [(1 + \rho)^r - 1]\right) [(1 + \rho)^r - 1 - r\rho(1 + \rho)^{r-1}] \quad (27)$$

B. V-BLAST

The performance of V-BLAST detection at finite SNR depends highly on the receiver processing technique. The most simple processing technique is zero-forcing [11] where the channel inversion is performed. An example of the V-BLAST detection algorithm is given in Algorithm 1. It is well known [1], [3] that the post-processing SNR of each stream is not identical. The work in [10] focuses on simultaneous detection, and therefore we note that the diversity order of the stream does not increase with each detection stream. Under this assumption, the outage probability is dominated by the weakest stream's post-processing SNR. The resulting outage probability is given as [10]

$$P_{out}(r, \rho) = \Pr[C_v(t) < r \log_2(1 + \rho)] \quad (28)$$

where $C_v(t) = M_T \log_2(1 + \min_{i \in \{1, \dots, M_T\}} \eta_i)$ and η_i is given as

$$\eta_i = \frac{\rho}{M_T [(\mathbf{H}^\dagger \mathbf{H})^{-1}]_{i,i}}, i = 1, \dots, M_T \quad (29)$$

where $[\cdot]_{i,j}$ denotes the (i, j) th element of the corresponding matrix. In [10], bounds are placed on the resulting outage probability. The upper bound is given as

$$P_{outU} = \frac{\Gamma(\frac{M_T^2}{\rho} [(1 + \rho)^{r/M_T} - 1], M_R - M_T + 1)}{(M_R - M_T)!} \quad (30)$$

$$d_{LB}(r, \rho) = \frac{\frac{M_T}{\rho} \left(\frac{M_T}{\rho} X\right)^{M_T - M_R}}{\Gamma\left(\frac{M_T}{\rho} X, M_T - M_R + 1\right)} \exp\left(\frac{-M_T}{\rho} X\right) \left[X - \frac{r\rho}{M_T} (1 + \rho)^{r/M_T - 1}\right], \quad X = (1 + \rho)^{r/M_T} - 1 \quad (33)$$

$$d_{UB}(r, \rho) = \frac{\frac{M_T^2}{\rho} \left(\frac{M_T^2}{\rho} X\right)^{M_T - M_R}}{\Gamma\left(\frac{M_T^2}{\rho} X, M_T - M_R + 1\right)} \exp\left(\frac{-M_T^2}{\rho} X\right) \left[X - \frac{r\rho}{M_T} (1 + \rho)^{r/M_T - 1}\right], \quad X = (1 + \rho)^{r/M_T} - 1 \quad (34)$$

and the lower bound as

$$P_{outL} = \frac{\Gamma\left(\frac{M_T}{\rho} [(1 + \rho)^{r/M_T} - 1], M_R - M_T + 1\right)}{(M_R - M_T)!} \quad (31)$$

Finally using (22), (30) and (31) one can obtain the bounded diversity-multiplexing tradeoff for the V-BLAST architecture as

$$d_{LB}(r, \rho) \leq d_V(r, \rho) \leq d_{UB}(r, \rho) \quad (32)$$

where $d_{LB}(r, \rho)$ and $d_{UB}(r, \rho)$ are given in (33) and (34) respectively.

C. Observations

Figures 8 and 9 show the finite SNR tradeoffs of the Alamouti and V-BLAST schemes respectively. From these we draw several important conclusions. Firstly we note that the required SNR in order to approximate the infinite SNR performance is quite high for both transmission schemes ($> 50dB$). This means that the optimal tradeoff Alamouti and V-BLAST tradeoff derived by Zheng is limited in the sense that it largely varies under practical SNR values.

For the case of the V-BLAST detection, we also note that for low the SNR region, the upper and lower bounds on the DIV-MUX tradeoff are quite large and in fact for low SNR the full multiplexing gain is not easily exploited. The upper and lower bounds converge for high SNR.

VI. CONCLUSION

In this project we have examined the optimal diversity-multiplexing tradeoff of MIMO systems under both Rayleigh i.i.d. conditions and a more general MIMO channel model. Graphs showing these resulting tradeoffs as well as the tradeoffs of the Alamouti and BLAST schemes were generated for both the infinite and finite SNR diversity-multiplexing tradeoff. Results, restrictions and complexities concerns were briefly analyzed and conclusions drawn as well as a simple V-BLAST detection algorithm was described..

A. Future Work

In future work we will examine the actual stream bit error rates such as those examined by Loyka [12], [13] to determine the error rate performance of the V-BLAST detection scheme for implementation.

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